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# A NOTE ON CONSTRUCTION OF BIB DESIGNS

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SUMMARY

A method of construction of BIB designs is given. This leads to a new series of BIB designs.

*Keywords* : Balanced incomplete blocks; $\alpha$-resolvable designs.

## Introduction

If the blocks of a Balanced Incomplete Block (BIB) design with parameters $v, b, r, k, \lambda$ can be grouped into $t$ groups of $\beta$ blocks each, such that each group contains each treatment exactly $\alpha$ times then the design is said to be $\alpha$–resolvable. Further if any pair of blocks belonging to the same group contains $q_1$ treatments in common where as any pair of blocks from different groups contain $q_2$ treatments in common, then the design is said to be affine $\alpha$–resolvable. For an affine $\alpha$-resolvable BIB design we have necessarily (Kageyama, 1973) : $b = \beta t$, $r = \alpha t$, $v\alpha = \beta k$, $q_1 = (\alpha - 1)\, k/(\beta - 1) = k + \lambda - r$, $q_2 = \alpha k/\beta = k^2/v$. In this note a method of construction of BIB design is proposed. Applying this method to affine $\alpha$-resolvable BIB designs for $\alpha > 1$ a new series of BIB design is obtained.

## 2. Main Result

THEOREM. *The existence of an affine $\alpha$-resolvable BIB design with parameters $v, b = \beta t$, $r = \alpha t$, $k, \lambda$, where $q_1 = k(\alpha - 1)/\beta - 1 \geqslant 2$, $\alpha = \beta - 1$, $k \geqslant 2\alpha/\alpha - 1$, $(\alpha > 1)$, implies the existence of a BIB design with* *parameters* $v^*$, $b^*$, $r^*$, $k^*$ *and* $\lambda^*$ *where* $v^* = v$, $b^* = t\binom{\beta}{2}$, $r^* = \binom{\alpha}{2}$, $k^* = k(\alpha - 1)/\alpha$, $\lambda^* = t_1\binom{\alpha}{2} + t_2\binom{\alpha-1}{2}$ *where* $t_1 = \lambda - t(\alpha - 1)$, $t_2 = t\alpha - \lambda$. *The new design is constructed by forming blocks from the pairwise intersection of blocks belonging to the same group of the given affine* $\alpha$*-resolvable BIB design.*

*Proof.* In any affine $\alpha$-resolvable BIB design any pair of blocks of a particular group have $k(\alpha - 1)/(\beta - 1)$ treatments common. Under the condition $(\beta - 1) = \alpha$, $k(\alpha - 1)/(\beta - 1) = k(\alpha - 1)/\alpha$ is the number of treatments which will form a block. Therefore it is obvious that $v^* = v$ and $k^* = k(\alpha - 1)/\alpha$. There are $\beta$ blocks in each group and for $t$ groups the total number of blocks will be $t\binom{\beta}{2}$ and hence $b^* = t\binom{\beta}{2}$. In each group each treatment occurs $\binom{\alpha}{2}$ times and for $t$ groups the total number of replications is $t\binom{\alpha}{2}$ and hence $r^* = t\binom{\alpha}{2}$. Since $\beta = \alpha + 1$ there are $t_1$ groups in which a pair of treatments occurs $\alpha$ times and $t_2$ groups in which a pair of treatments occurs $(\alpha - 1)$ times where $t_1 + t_2 = t$, and $t_1\alpha + t_2(\alpha - 1) = \lambda$. Hence the number of times that a pair of treatments occurs in the new design is $\lambda^* = t_1\binom{\alpha}{2} + t_2\binom{\alpha-1}{2}$ where $t_1$ and $t_2$ are given by the expressions above. This completes the proof.

For example consider the affine 3-resolvable BIB design with parameters $v = 16$, $b = 20$, $r = 15$, $k = 12$, $\lambda = 11$, whose blocks are as follows :

(5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16); (1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16); (1, 2, 3, 4, 5, 6, 7, 8, 13, 14, 15, 16); (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12); (2, 3, 4, 6, 7, 8, 10, 11, 12, 14, 15, 16); (1, 3, 4, 5, 7, 8, 9, 11, 12, 13, 15, 16); (1, 2, 4, 5, 6, 8, 9, 10, 12, 13, 14, 16); (1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15); (2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15); (1, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 15); (1, 2, 4, 5, 6, 7, 10, 11, 12, 13, 15, 16); (1, 2, 3, 5, 6, 8, 9, 11, 12, 14, 15, 16); (2, 3, 4, 5, 6, 8, 9, 10, 11, 13, 15, 16); (1, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 16); (1, 2, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15); (1, 2, 3, 5, 7, 8, 10, 11, 12, 13, 14, 16); (2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16); (1, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15); (1, 2, 4, 5, 7, 8, 9, 10, 11, 14, 15, 16); 1, 2, 3, 6, 7, 8, 9, 10, 12, 13, 15, 16).

Hence $t = 4$, $t_1 = 1$, $t_2 = 4$, $\alpha = 3$ and $\beta = 4$.

The parameters of the BIB design derived by the given method are $v^* = 16$, $b^* = 30$, $r^* = 15$, $K^* = 8$ and $\lambda^* = 7$. The blocks of this design are given below.

(9, 10, 11, 12, 13, 14, 15, 16); (5, 6, 7, 8, 13, 14, 15, 16); (5, 6, 7, 8, 9, 10, 11, 12); (1, 2, 3, 4, 13, 14, 15, 16); (1, 2, 3, 4, 9, 10, 11, 12); (1, 2, 3, 4, 5, 6, 7, 8); (3, 4, 7, 8, 11, 12, 15, 16); (2, 4, 6, 8, 10, 12, 14, 16); (2, 3, 6, 7, 10, 11, 14, 15); (1, 4, 5, 8, 9, 12, 13, 16); (1, 3, 5, 7, 9, 11, 13, 15);

(1, 2, 5, 6, 9, 10, 13, 14); (3, 4, 7, 8, 9, 10, 13, 14); (2, 4, 5, 7, 10, 12, 13, 15); (2, 3, 5, 8, 9, 12, 14, 15); (1, 4, 6, 7, 10, 11, 13, 16); (1, 3, 6, 8, 9, 11, 14, 16); (1, 2, 5, 6, 11, 12, 15, 16); (3, 4, 5, 6, 9, 10, 15, 16); (2, 4, 6, 8, 9, 11, 13, 15); (2, 3, 5, 8, 10, 11, 13, 16); (1, 4, 6, 7, 9, 12, 14, 15); (1, 3, 5, 7, 10, 12, 14, 16); (1, 2, 7, 8, 11, 12, 13, 14); (3, 4, 5, 6, 11, 12, 13, 14); (2, 4, 5, 7, 9, 11, 14, 16); (2, 3, 6, 7, 9, 12, 13, 16); (1, 4, 5, 8, 10, 11, 14, 15); (1, 3, 6, 8, 10, 12, 13, 15); (1, 2, 7, 8, 9, 10, 15, 16).

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## REFERENCES

[1] Bose, R. C. (1942) : A note on the resolvability of balanced incomplete block designs, *Sankhya*, 6 : 105-110.

[2] Kageyama, S. (1973) : On μ-resolvable and affine μ-resolvable balanced incomplete block designs, *Ann. Statist.*, 1 : 195-203.

[3] Kageyama, S. (1973) : A series of three designs, *J. Japan Statist. Soc.*, 3 : 67-68